# The secant method

1. Use three steps of the secant method to approximate a root of the function $f(x) \stackrel{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ starting with $x_0 = 3.0$ and $x_1 = 3.2$.

Answer: To ten significant digits, we have 3.0, 3.2, 3.260614340, 3.266354278, 3.266500105

2. Use three steps of the secant method to approximate a root of the function $f(x) \stackrel{\text{def}}{=} x^3 - 3x + 1$ starting with $x_0 = -1.5$ and $x_1 = 1.3172$.

Answer: To ten significant digits, we have –1.5, 1.3172, 0.6447658186, 27376.62957, 0.6447658195.

3. What is the cause for the sequence of approximations in Question 2?

Answer: First, $f(x_1)$ approximately equals $f(x_2)$, so the denominator is very small, but next, $f(x_3)$ is huge, and therefore $x_4$ ends up being very close to $x_2$ again.

4. If you continue to iterate the secant method in Question 2, what root does it converge to?

Answer: To ten significant digits, 0.3472963553

5. If you iterated the secant method in Question 2 but starting with $x_1 = 1.4$, what root does it converge to?

Answer: To ten significant digits, 1.532088886.

6. In general, should you apply the secant method if you don't already have an idea as to what a root of a function is?

Answer: In general, no. The secant method is a tool to refine an approximation of a root, not to check if a function has a root. If you start with an arbitrary initial point, it may or may not converge to a root if there is one, so non-convergence does not suggest there is no root.

7. The function $x^2$ has a double root at $x = 0$. Apply the secant method starting with the initial value $x_0 = 1$ and $x_1 = 0.5$. Is the convergence still $O(h^{\phi})$?

Answer: No, with a double root, the rate of convergence becomes $O(h)$ and if you examine this example closely, the error drops by $h/\phi$ where $\phi$ once again is the golden ratio.